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Preconditioned dual-time procedures and its application to simulating the flow with cavitations

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Abstract

Preconditioning technology is a method that can unify the method of CFD for high-speed, low-speed, compressible and incompressible flows. In the flow with cavitations, incompressible liquid and compressible vapor coexist. The phase transition in the flow results in complex thermophysical processes. In this paper, based on the primitive variables pressure, velocity and enthalpy, a preconditioned dual-time procedures is established, which can deal with unsteady flow field with cavitations. The energy equation is adopted to replace the traditional cavitational model. Roe's scheme is used to capture the disconnected surface. The examples indicate that this method can be used to capture the unsteady cavity in the flow field.

Keywords: Unsteady flow; Cavitations; Equation of state

1. Introduction

Preconditioning technology, which is a method that can unify the method of CFD for high-speed, low-speed, compressible and incompressible flows, has been widely used [1–4]. For the steady flow problem, time-marching algorithms are used to solve the flow field by introducing the pseudo-time steps. Nowadays, most preconditioning technologies use pressure, velocity and temperature as its primal variables [1–5]. Investigation indicates that this method is efficient to deal with flow fields without phase transition. But for the flow field with phase transition, in the phase transition region, the saturation pressure is a function of temperature only, and is independent of density, so it is unjustifiable to use pressure and temperature as independent variables to solve flow problems with cavitations. A new preconditioning technology using pressure, velocity and enthalpy as its primal variables was reported by Huang [6].

For unsteady flows, a dual-time procedures is adopted, of which, the physical time layer is used to track the physical variety of the flow, while a pseudo-time layer is used to iterate every physical time step [7,8]. In this paper, we try to apply the dual-time procedures into the new preconditioning technology based on the primal variables of pressure, velocity and enthalpy, to solve unsteady flows with cavitations.

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2. Fundamental equations

A new preconditioning technology, based on conservational Reynolds averaged Navier–Stokes equations, with pressure, velocity and enthalpy (p,u,v,w,h) as its primal variables, was constructed in Ref. [6]

$$\mathbf{\Gamma}\frac{\partial \boldsymbol{Q}}{\partial \tau}+\frac{\partial \boldsymbol{E}}{\partial \boldsymbol{Q}}\frac{\partial \boldsymbol{Q}}{\partial \xi}+\frac{\partial \boldsymbol{F}}{\partial \boldsymbol{Q}}\frac{\partial \boldsymbol{Q}}{\partial \eta}+\frac{\partial \boldsymbol{G}}{\partial \boldsymbol{Q}}\frac{\partial \boldsymbol{Q}}{\partial \zeta}=\frac{\partial \boldsymbol{E}^{\mathrm{v}}}{\partial \boldsymbol{Q}}\frac{\partial \boldsymbol{Q}}{\partial \xi}+\frac{\partial \boldsymbol{F}^{\mathrm{v}}}{\partial \boldsymbol{Q}}\frac{\partial \boldsymbol{Q}}{\partial \eta}+\frac{\partial \boldsymbol{G}^{\mathrm{v}}}{\partial \boldsymbol{Q}}\frac{\partial \boldsymbol{Q}}{\partial \zeta}. \tag{1}$$

Of which, $\boldsymbol{Q}=\boldsymbol{J}^{-1}(p,u,v,w,h)$, $\mathbf{\Gamma}$ is the preconditioning matrix, $\boldsymbol{E}$, $\boldsymbol{F}$ and $\boldsymbol{G}$ are the inviscid fluxes in the ξ, η, ζ coordinate directions, respectively. $\boldsymbol{E}^{\mathrm{v}}$, $\boldsymbol{F}^{\mathrm{v}}$ and $\boldsymbol{G}^{\mathrm{v}}$ are the viscous fluxes in the ξ, η, ζ coordinate directions, respectively. The detailed description of $\boldsymbol{E}$, $\boldsymbol{F}$, $\boldsymbol{G}$, $\boldsymbol{E}^{\mathrm{v}}$, $\boldsymbol{F}^{\mathrm{v}}$ and $\boldsymbol{G}^{\mathrm{v}}$ can be found in many books. The preconditioning matrix $\mathbf{\Gamma}$ can be described as follows:

$$\mathbf{\Gamma}=\begin{bmatrix} \Theta & 0 & 0 & 0 & \frac{\partial\rho}{\partial h} \\ u\Theta & \rho & 0 & 0 & u\frac{\partial\rho}{\partial h} \\ v\Theta & 0 & \rho & 0 & v\frac{\partial\rho}{\partial h} \\ w\Theta & 0 & 0 & \rho & w\frac{\partial\rho}{\partial h} \\ H\Theta-1 & \rho u & \rho v & \rho w & H\frac{\partial\rho}{\partial h}+\rho \end{bmatrix},$$

where H is total enthalpy, $\Theta=\frac{1}{U_r^2}-\frac{\partial\rho}{\rho\partial h}$, and

$$U_r=\begin{cases} \varepsilon c & \text{if } U'<\varepsilon c, \\ U' & \text{if } \varepsilon c<U'<c, \\ c & \text{if } U'>c, \end{cases} \quad \varepsilon=10^{-5}.$$

It has been proved that Eq. (1) cannot be singular, whatever for high-speed or low-speed flows, so the time-matching method is always suitable to be adopted to solve the equations.

For steady flows, τ denotes the pseudo-time layer. The convective terms of Eq. (1) are discretized by Roe's scheme. The diffusion terms are discretized by the central difference scheme. An implicit pseudo-time marching approach, a first-order difference scheme, is used to get the steady solutions.

$$\mathbf{\Gamma}\frac{\Delta\boldsymbol{Q}^{k+1}-\Delta\boldsymbol{Q}^{k}}{\Delta\tau}+\delta_\xi\boldsymbol{E}^{k+1}+\delta_\eta\boldsymbol{F}^{k+1}+\delta_\zeta\boldsymbol{G}^{k+1}=\boldsymbol{R}_{hs} \tag{2}$$

here,

$$\delta_\xi\boldsymbol{E}^{k+1}=\delta_\xi\boldsymbol{E}^{k}+\boldsymbol{A}^{k}\delta_\xi(\Delta\boldsymbol{Q}^{k+1}),$$

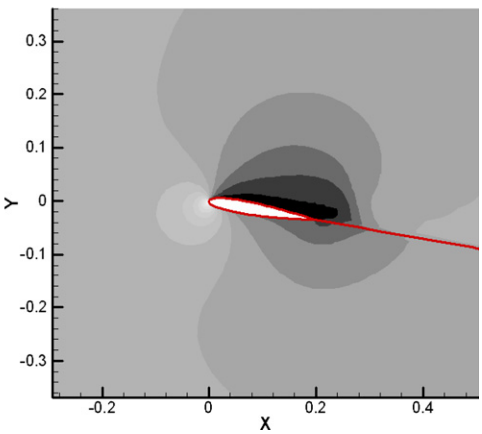

Fig. 1. The contour lines of pressure around NACA0012 wing at the attack angle of 10°.

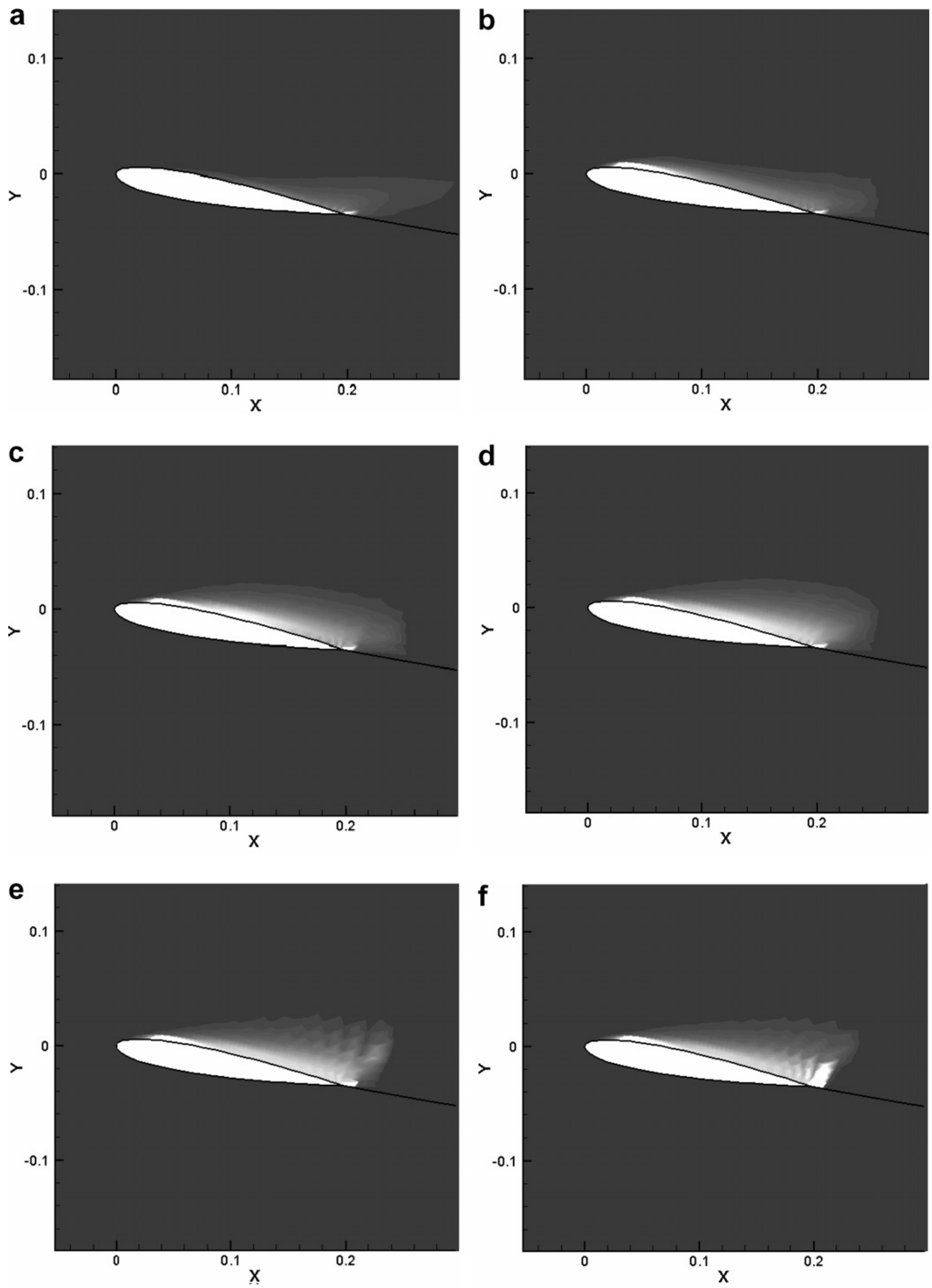

Fig. 2. The contour lines of specific volume around the wing at various time steps. Except in the wing shape, the cavitating region (vapor) is whitest, the darkest area is water, grayer area is cloud region. (a) is at 0.05 s. from this figure, we can see a small cavitation region occurring near the tail of the wing, and there is also a piece of cloud around the cavitation. (b) is at 0.15 s, this figure shows that the cavitating region increases obviously, the cavitating region occurs all over the upper surface of the wing except near the head. The cloud around the cavitation also increases obviously. (c) and (d) are at 0.30 s and 0.40 s respectively. The cavitation increases from a sheet cavitation to a bubble continuously as time elapses, and the cloud around the cavitation also increases continuously. (e) and (f) are at 0.55–0.65 s, respectively. The shape of cavitation and cloud barely changes. (g) and (h) are at 0.66–0.67 s. The cavitation begins to crack near the tail of the cavitation. (i) is at 0.70 s, the cavitation recovers a sheet after cracking.

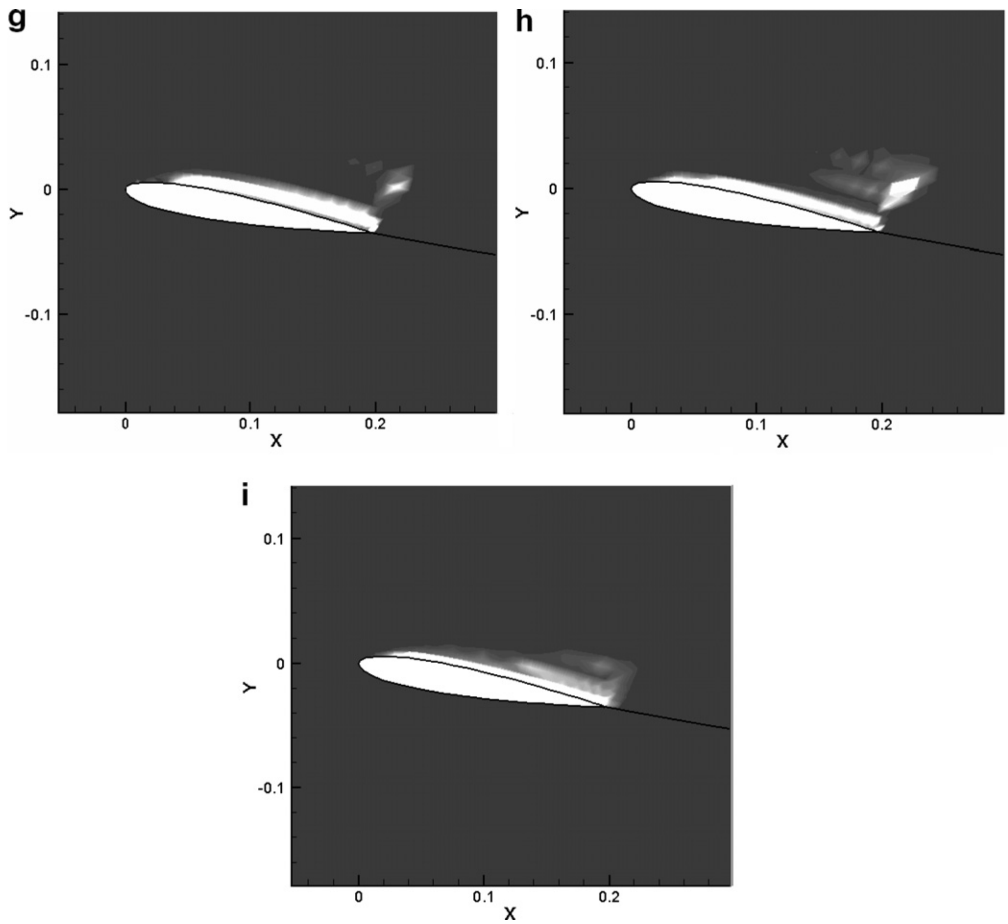

Fig. 2 (*continued*)

$$\delta_\eta \boldsymbol{F}^{k+1} = \delta_\eta \boldsymbol{F}^{k} + \boldsymbol{B}^{k}\delta_\eta(\Delta \boldsymbol{Q}^{k+1}), \quad \delta_\zeta \boldsymbol{G}^{k+1} = \delta_\zeta \boldsymbol{G}^{k} + C^{k}\delta_\zeta(\Delta \boldsymbol{Q}^{k+1}),$$

$$\boldsymbol{R}_{hs} = \boldsymbol{\Gamma}\frac{\Delta \boldsymbol{Q}^{k}}{\Delta\tau} + (\boldsymbol{E}^{\mathrm{v}} - \boldsymbol{E}^{\mathrm{v}} + \boldsymbol{F}^{\mathrm{v}} - \boldsymbol{F}^{\mathrm{v}} + \boldsymbol{G}^{\mathrm{v}} - \boldsymbol{G}^{\mathrm{v}})^{k},$$

where δ denotes the spatial difference, and the subscripts denote the differential directions. A, B, C denote the flux Jacobian matrices corresponding to the primal variables Q, respectively. k is the index of the pseudo-time step.

For unsteady flows, a physical time layer is needed. Discretizing the physical time layer using a implicit second-order difference scheme

$$\boldsymbol{\Gamma}\frac{\Delta \boldsymbol{Q}^{k+1}}{\Delta\tau} + \frac{3\boldsymbol{W}^{k+1} - 4\boldsymbol{W}^{n} + \boldsymbol{W}^{n-1}}{2\Delta \boldsymbol{t}} + \delta_\xi \boldsymbol{E}^{k+1} + \delta_\eta \boldsymbol{F}^{k+1} + \delta_\zeta \boldsymbol{G}^{k+1} = \boldsymbol{R}_{hs} \tag{3}$$

of which, $\boldsymbol{W}$ is the conservational variable. $\boldsymbol{W} = (\rho, \rho u, \rho v, \rho w, \rho e)$, n is the index of the physical time step. $\boldsymbol{W}^{k+1} = \boldsymbol{W}^{k} + (\partial \boldsymbol{W}/\partial \boldsymbol{Q})\Delta \boldsymbol{Q}^{k+1}$. So, the discrete form of (3) is given by

$$\left[\boldsymbol{\Gamma} + (\partial \boldsymbol{W}/\partial \boldsymbol{Q})\frac{3\Delta\tau}{2\Delta t} + \Delta\tau(\boldsymbol{A}\delta_\xi + \boldsymbol{B}\delta_\eta + \boldsymbol{C}\delta_\zeta)\right]\Delta \boldsymbol{Q}^{k+1} = \boldsymbol{R}^{k}, \tag{4}$$

where

$$\boldsymbol{R}^{k} = \boldsymbol{R}_{hs} - \Delta\tau\left(\frac{3\boldsymbol{W}^{k} - 4\boldsymbol{W}^{n} + \boldsymbol{W}^{n-1}}{2\Delta t}\right) - \Delta\tau(\delta_\xi \boldsymbol{E}^{k} + \delta_\eta \boldsymbol{F}^{k} + \delta_\zeta \boldsymbol{G}^{k}).$$

For the above equation system, the primal variables ΔQ^{k+1} at each pseudo-time step can be solved by a straightforward block tri-diagonal solver line by line.

3. Examples

Consider water flow over the NACA0012 wing at an angle of attack of 10° for example. The cavitation number $\sigma = 0.48$, the physical time step is 0.01 s. Fig. 1 is the contour lines of pressure at 0.2 s. The blackest region in the field above the wing is the region where the pressure has reached the saturation pressure, so it is in the region of cavitations. Fig. 2 is the contour lines of specific volume around the wing at various time steps. These figures indicate the process of evolvement of unsteady cavitation around the wing.

4. Conclusions

A dual-time procedures is implemented into a new preconditioning technology based on the primal variables of pressure, velocity and enthalpy. Of which, the physical time layer is used to track the physical variety of the flow, while the pseudo-time layer is used to iterate every physical time step. The primary calculations indicate that the method we introduced in this paper for unsteady flows with cavitations can capture the unsteady cavity.

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